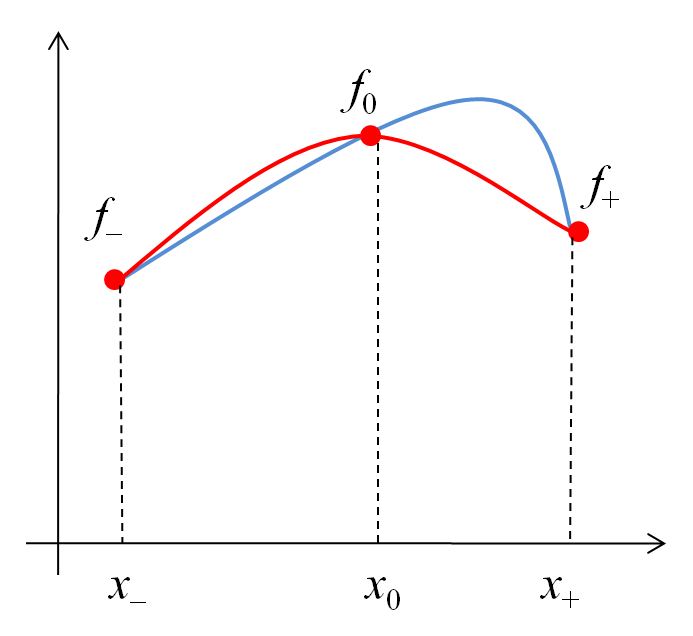
Simpson’s Rule on a Generic Mesh

The classic form of Simpson’s Rule is defined on a (uniform) mesh, by virtue of the factor of two in the midpoint divisor.

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An interesting strategy is that

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Is exactly equivalent to I = y(b) where y(x) is the solution of the differential equation:

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With the boundary condition that y(a) = 0.

Applying the “centered approximation”:

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Here we have defined *h*+ = (*xi* + 1 – *xi*) and *h*– = (*xi* – *xi* – 1).

Substitute f(x) for y’

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Derivatives of f are

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Substituting:

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Collecting terms on F0:

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So the coefficient on F0 reduces to (!)

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Now collecting terms on f-

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Now collecting on f+

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Substituting:

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When a = b, this reduces to:

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And I believe this is correct, given that the range here corresponds to a = 0, b = 2a in Simpson’s formula:

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Calculating y1

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In our case,

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Using the definition of f utilized earlier in Eq 6,

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Similarly,

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And substituting:

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Which is just the trapezoidal rule. Using the boundary condition that y0 = 0,

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Using a one-sided approximation for the second derivative of f,

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Note that b is the distance between u++ and u+, h is the distance between u+ and u0.

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